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EXPERIMENTAL INVESTIGATIONS OF THE COMPRESSIBILITY OF SANDY SOILS AND THE CONDITION OF PLASTICITY DURING BRIEF DYNAMIC LOADINGS

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§1. Experimental investigations of samples of sandy soils with a bulk weight of $\gamma_0=1.50$ g cm^3, moisture content w=0.003 (air-dried soil) and 0.05; 0.15 were carried out on a quasistatic type of equipment, similar in construction to that described earlier in [1]. The equipment consists of a vertically standing cylinder with the sample of soil distributed in it in a special collar with a diameter of $D_0=150$ mm and height $h_0=30$ mm and a piston which transmits a shock loading to the soil sample. Different conditions of deformation of the samples were created by means of rubber spacers and by varying the drop height of the load. In addition, static tests of the samples were carried out at a rate of deformation of $\varepsilon=2\cdot10^{-3}$ to $0.5\cdot10^{-5}$ sec^{-1}. Each sample was subjected to a triple loading. A fivefold repeat of the experiments was provided for, under one and the same conditions (series of experiments).

The principal stresses in the sample $\sigma_1(t)$ and $\sigma_2(t)$ were recorded by means of tensometric probes, installed in the center of the piston, in the center and edge of the cylinder bottom (four probes), and in the lateral surface of the collar (two probes). The total force transmitted to the sample by the shock was recorded also by means of a tensometric thimble.

The tensometric probes for measuring the stresses had a diameter of the sensitive element (a round thin plate, pinched around the outline) of d = 22 mm and a thickness of $\delta=2$ to 4 mm. The systematic errors of the stress measurements with these probes in the range of loads investigated were studied in [3] and did not exceed ±3 to 5% in the experiments.

Comparison of the readings of the probes located in the center of the piston and in the center and edge of the cylinder bottom confirms their agreement with an accuracy up to the random measurement errors. A similar conclusion can be drawn with respect to the data on the stresses $\sigma_1(t)$ obtained by measurements of the total force transmitted to the sample by the shock. Therefore, in the future, all probes for measuring the stresses $\sigma_1(t)$ will be treated as equally justified. It was assumed, therefore, that the stresses over the height of the sample and over its diameter are distributed uniformly.

The deformations of the sample were measured by means of three tensometric displacement probes, positioned at angles of 120° (in the plane of the sample). The displacement probe consisted of two arms, rigidly attached in the lower part of the cylinder. A wedge, joined to the movable piston of the equipment, was installed between the arms. Tensometers were secured to the arms, the signals from which are proportional to the displacement of the piston.

The deformation was determined in quasistatic approximation by the relation $\varepsilon(t)=u(t)/h_0$, where u(t) is the displacement of the piston. The stresses to be recorded $\sigma_{1i}(t)$ and $\sigma_{2i}(t)$ and the deformation $\varepsilon_i(t)$ (i =

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TABLE 1

w	No. of land-ing	k	r	r_1	r_2	ξ
	1	0,893	0,664	—0,106	0,912	0,550
0,003	2	1,183	0,881	0,421	0,970	0,461
	3	1,200	0,567	—0,799	0,975	0,455
	1	1,493	0,972	0,831	0,992	0,379
0,05	2	1,595	0,942	0,674	0,986	0,354
	3	1,649	0,979	0,869	0,995	0,341
	1	1,559	0,914	0,549	0,979	0,361
0,15	2	1,763	0,960	0,761	0,990	0,320
	3	1,782	0,945	0,790	0,991	0,315

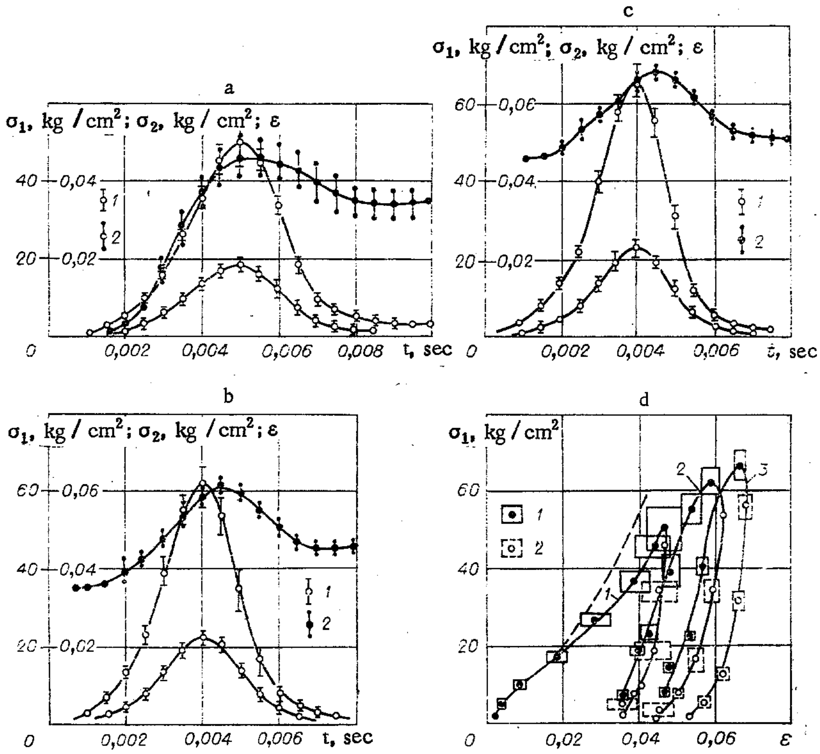

Fig. 1

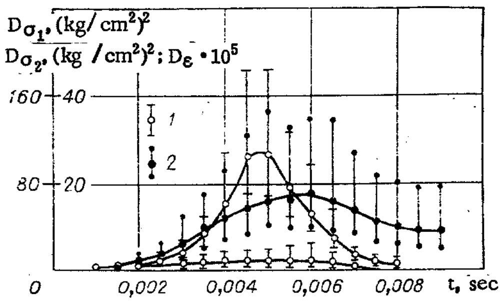

Fig. 2

1, 2,..., l; is the number of measurements in the selection) represent certain completions of random processes. In processing the results, a set of these completions for fixed instants of time t_j (j = 1, 2, ..., n) was considered as a system of random quantities. Here n is the number of equal intervals Δt into which the complete process was divided for each of the probes in the experiments of a given series. The size of the intervals Δt was $\Delta t = (0.25 \text{ to } 1.0) \cdot 10^{-3}$ sec, depending on the duration of the loading in the series. For each of the instants t_j (j = 1, 2, ..., n) in the series of experiments, verification was carried out of the hypothesis concerning the normal distribution of the stresses $\sigma_1(t_j)$, $\sigma_2(t_j)$ and the deformations $\varepsilon(t_j)$ by means of the Wilkes W-criterion [4]. Then for each of the random quantities $\sigma_1(t_j)$, $\sigma_2(t_j)$, and $\varepsilon(t_j)$ (j = 1, 2, ..., n) an estimate was undertaken of the distribution parameters (mathematical expectation and dispersion) by the method of confidence intervals, by means of the Student distribution and the χ^2 distribution, respectively [5, 6].

If the calculated value of W was found to be less than 5% of the critical value, assumed from the table for a given selection, then the probability that the selection is taken from a set distributed according to a normal law did not exceed 0.05. In these cases, the hypothesis of normal distribution must be abandoned.

By using the Gram–Charles [5] series for representing the law of distribution of a random quantity, it can be shown that in the case of small asymmetry ($A \leq 0.5$ to 0.8) the distribution can be taken as approximately normal. When $A > 0.5$ to 0.8, the distribution differs significantly from normal. When processing the experiments, $A = 0.7$ was assumed for the criterion.

It should be noted that the latter circumstance is significant only for estimating the accuracy of determination of the dispersion. An estimate of the mathematical expectation in these experiments can be carried out in all cases with sufficient accuracy on the basis of the law of normal distribution of the arithmetic mean because of the adequate volume of the selection [5, 6].

An estimate of the confidence intervals for linear regression when determining the condition for plasticity was carried out by means of the Student distribution, and when estimating the confidence intervals for the correlation coefficients the Fisher transformation [5] was used.

§2. Figure 1a-c shows the measurement results of the stresses $\sigma_1(t)$, $\sigma_2(t)$ and the deformations $\varepsilon(t)$ for three successive loadings of samples of sandy soil with $\gamma_0 = 1.50$ g/cm^3, w = 0.05. Figure 1a-c corresponds to the first, second, and third loadings. The symbols are as follows: 1) the arithmetic mean of the stress values $\sigma_1(t_j) = \frac{1}{l}\sum_{i=1}^{l}\sigma_{1i}(t_j)$, $\sigma_2(t_j) = \frac{1}{l}\sum_{i=1}^{l}\sigma_{2i}(t_j)$; 2) deformations $\varepsilon(t_j) = \frac{1}{l}\sum_{i=1}^{l}\varepsilon_i(t_j)$ with the corresponding confidence intervals $\pm I_\beta$, determined with a reliability of $\beta = 0.9$.

The relative confidence intervals for the arithmetic means $\alpha(\sigma_{1j}) = \pm I_\beta(\sigma_{1j})/\sigma_{1j}$, $\alpha(\sigma_{2j}) = \pm I_\beta(\sigma_{2j})/\sigma_{2j}$, and $\alpha(\varepsilon_j) = \pm I_\beta(\varepsilon_j)/\varepsilon_j$, as follows from the data of Fig. 1a-c, amount to $\alpha = \pm(0.10$ to $0.15)$ when $0.0025 \leq t \leq 0.0075$ sec for the stresses and when $0.003 \leq t \leq 0.010$ sec for the deformations.

At the initial and final instants (in the latter case for the stresses), the magnitudes of α can be greater because of the impossibility of guaranteeing identical measurement accuracy for small and large values of the quantities being measured by means of one and the same probe.

Figure 2 shows the results of an estimate of the stress dispersions $D_{\sigma_1}(t_j)$, $D_{\sigma_2}(t_j)$ and the deformations $D_\varepsilon(t_j)$, corresponding to the data of Fig. 1. The symbols 1 and 2 are the estimates of the stress and deformation dispersions, with the corresponding confidence intervals, determined with a reliability of $\beta = 0.9$. It follows from Fig. 2 that for the main part of the process the coefficients of variation for stresses and deformations amount to $k_\nu = (0.20$ to $0.30)$.

Thus, a series of five experiments permitted the experimental accuracy to be increased significantly (up to a factor of two). Estimates of the corresponding distribution parameters for other cases are similar. A somewhat lower accuracy was obtained for air-dried soil.

The hypothesis concerning the law of normal distribution for the quantities $\sigma_1(t_j)$, $\sigma_2(t_j)$, and $\varepsilon(t_j)$ being measured, as shown by the estimates by means of W, cannot be abandoned for the main part of the process for the soils investigated under different conditions of deformation. Exceptions in a number of cases are the initial and final instants of time, where the measurement accuracy is low.

In Fig. 1d, the curves of $\sigma_1(\varepsilon)$ are plotted according to the data of Fig. 1a-c by an exclusion method of the time t for the first, second, and third loadings (curves 1-3, respectively). The points 1 correspond to the loading $(\partial\sigma_1/\partial t > 0)$, and the points 2 correspond to the stress relief $(\partial\sigma_1/\partial t < 0)$. It can be seen from these results that the curves of $\sigma_1(\varepsilon)$ in the case of repeated loadings do not coincide with the lines of the previous loading. The fraction of residual deformations ε_0 in relation to the maximum ε_* decreases with repeated loadings: $\varepsilon_0/\varepsilon_* = 0.72$ for the first loading and 0.44 and 0.25 for the second and third loadings, respectively.

In Fig. 1d, the dashed curve, obtained by using the laws of conservation at the shock front when $\dot{\varepsilon} = \infty$ [3, 7], corresponds to the results of field investigations in soil similar in characteristics ($\gamma_0 = 1.48$ g/cm^3 and $w = 0.05$).

The results given confirm with sufficient reliability the previously published data [1] concerning the important effect of the rate of deformation (viscosity) on the compressibility of sandy soils of natural moisture content. In this case, a significant effect was noted of the moisture content of the sandy soil on its viscoplastic properties.

In particular, for samples with $w = 0.003$ (air-driedsamples), within the limits of experimental accuracy in the case of maximum loadings up to 50 kg/cm^2 and with average rates of deformation $\dot{\varepsilon} \leq 17$-20 sec^{-1}, it was not possible to detect the rate of deformation effect on the compressibility of the soil. For samples with $w = 0.05$, as can be seen from Fig. 1d, this effect becomes significant.

Curves of $\sigma_1(t)$, $\sigma_2(t)$, and $\varepsilon(t)$ are shown in Fig. 3a-c, obtained for samples with $\gamma_0 = 1.50$ g/cm^3 and $w = 0.15$, for three loading conditions (the same as for the curves of Fig. 1.)

The $\sigma_1(\varepsilon)$ curves for these cases (1-3) are shown in Fig. 3d; the curve of $\sigma_1(\varepsilon)$, $\dot{\varepsilon} = \infty$, is shown dashed, as before, corresponding to the results of field investigations in similar soil ($\gamma_0 = 1.50$-1.52 g/cm^3 and $w = 0.12$-0.15), according to the data of [7]. Curve 4 corresponds to the data from static tests when $\dot{\varepsilon} = 0.5 \cdot 10^{-5}$ sec^{-1}. The difference in the deformations, corresponding to the limiting diagrams ($\dot{\varepsilon} = 0.5 \cdot 10^{-5}$ sec^{-1} and $\dot{\varepsilon} = \infty$) when $\sigma_1 = 20$-60 kg/cm^2, amounts to 300% in this soil. When $w = 0.05$, this difference is not more than 200% (Fig. 5). Thus, with increase of the moisture content up to $w = 0.15$, the role of the viscosity effects in sandy soil increased by comparison to soil with $w = 0.05$. The magnitude of the ratio $\varepsilon_0/\varepsilon_*$ when $w = 0.15$ for three successive loadings was reduced somewhat and amounted to 0.65, 0.37, and 0.24, respectively.

With increase of the duration of action, cases of a significant growth of deformation have been noted during stress relief ($\partial\sigma_1/\partial t < 0$). In particular, Fig. 4a, b shows the curves of $\sigma_1(t)$, $\sigma_2(t)$, and $\varepsilon(t)$ (Fig. 4a) and $\sigma_1(\varepsilon)$ (Fig. 4b) in the case of a single loading of sandy soil with $\gamma_0 = 1.50$ g/cm^3 and $w = 0.15$. In Fig. 4b, the deformations, when $\partial\sigma_1/\partial t < 0$, continue to grow during a certain time. With a shorter duration of the process, these effects were not observed for similar soil (Fig. 3d, curve 1).

Figure 5 shows curves of $\sigma_1(\varepsilon)$ for sandy soils with $\gamma_0 = 1.50$ g/cm^3 and with different moisture content [1) $w = 0.15$; 2) $w = 0.05$; 3) $w = 0.003$,] obtained for similar values of the average rates of deformation during loading [1) $\dot{\varepsilon}_1 = 13.0$; 2) $\dot{\varepsilon}_2 = 12.5$; 3) $\varepsilon_3 = 17.5$ sec^{-1}]. The confidence intervals are shown here only for the maximum values of the stresses and deformations. It can be seen that with an increase of moisture content from $w = 0.003$ to 0.15, deformation of the soil with values of $\sigma_1 = 20$-30 kg/cm^2 is significantly reduced (up to a factor of 1.75 when $\sigma_1 = 50$ kg/cm^2). Correspondingly, the form of the $\sigma_1(\varepsilon)$ curve is changed, at which there appears a clearly expressed point of inflection with increase of moisture content up to $w = 0.05$-0.15. When $w = 0.003$ and $\sigma_1 \leq 50$ kg/cm^2, the $\sigma_1(\varepsilon)$ curve has a convexity toward the stress axis ($\partial^2\sigma_1/\partial\varepsilon^2 < 0$). Similar data concerning the nature of the dynamic curves for sandy soils were obtained previously in [8].

§3. Figure 6 shows the results of investigations of the condition for plasticity for sandy soil with $\gamma_0 = 1.50$ g/cm^3 and $w = 0.05$. The quantity $T = \sqrt{2}(\sigma_1 - \sigma_2)$ is plotted along the axis of ordinates and the average stress $\sigma = (1/3)(\sigma_1 + 2\sigma_2)$ is plotted along the axis of abcissas. Each of the points in the plane (T, σ) is the mean arithmetic value for each of the experiments from the results of five measurements for $\sigma_1(t_j)$ and of two measurements for $\sigma_2(t_j)$, $j = 1, 2, \ldots, n$. The solid line is the line of linear regression of $T = k\sigma + b$, and the dashed line corresponds to the confidence interval for the linear regression with a reliability of $\beta = 0.9$. Similar results have been obtained for samples with a moisture content of $w = 0.003$ and 0.15. Table 1 shows data for the values of the coefficients k, the lateral pressure coefficients $\xi = (3\sqrt{2} - k)/(3\sqrt{2} + 2k)$ [1], and also the correlation coefficients r for the linear regression and the corresponding confidence intervals r_1, r_2 when $\beta = 0.9$ for the correlation coefficients for sandy soils of different moisture content. The values of b in all cases are equal to zero, with an accuracy up to the measurement error.

It can be seen from the table that with increase of moisture content, the values of the coefficients k are increased (correspondingly, the values of ξ are decreased) by a factor of 1.4 to 1.7. We note that the results obtained are close to the data obtained during tests of similar soils under field conditions [1, 7].

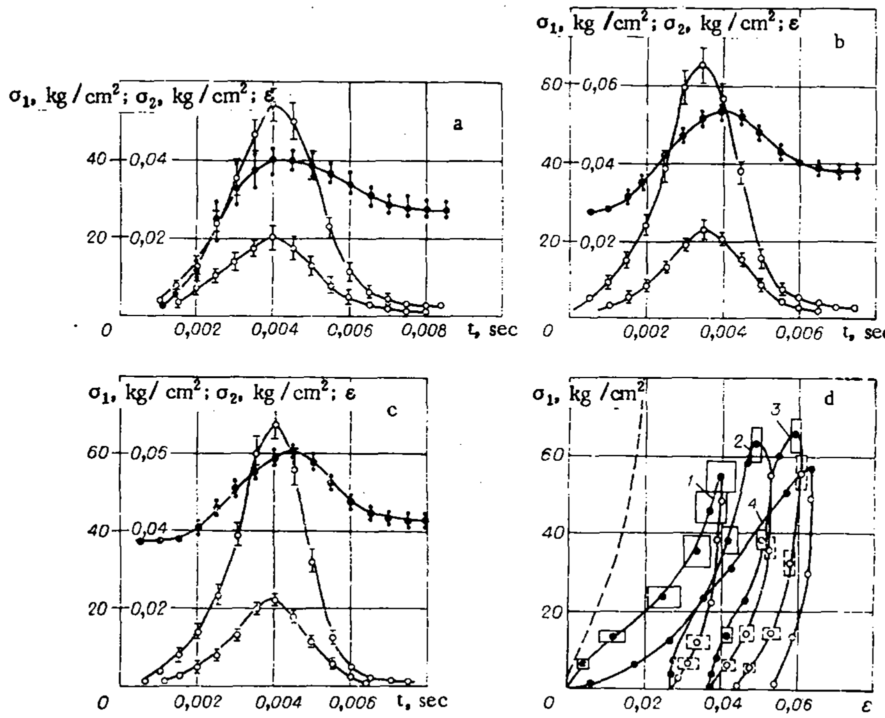

Fig. 3

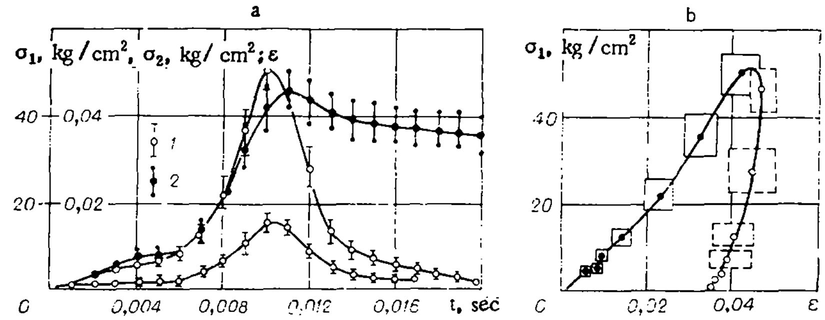

Fig. 4

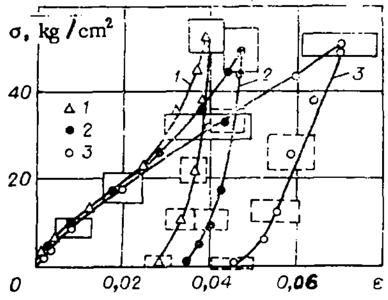

Fig. 5

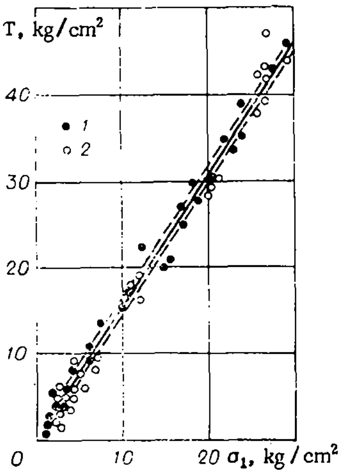

Fig. 6

It should be noted that for soil with w = 0.003 there is a considerably wider confidence interval for the correlation coefficient than for w = 0.05 to 0.15. Analysis of the results of the experiments shows that this fact is due to the marked difference of the function $T(\sigma)$ for air-dried soil for loading and stress-relief. For wet soils, the condition of plasticity (quantity k) within the limits of accuracy of the experiments is independent of the conditions of loading or stress-relief. In particular, the point 1 in Fig. 6 corresponds to loading ($\partial\sigma/\partial t > 0$) and point 2 corresponds to stress relief ($\partial\sigma/\partial t < 0$).

For conditions of static loading of sandy soil with w = 0.05, the value of k was found to be equal to 1.459, which also is close to the data of the table for the first loading.

With repeated loadings, the values of k increase somewhat. For air-dried soil (w = 0.003) this increase for three successive loadings amounts to 35% and for wet soil does not exceed 7-14%. The latter values are found to be within the limits of accuracy of the measurements.

It can be assumed, therefore, that for sandy soils of natural moisture content, the value of k does not change as a result of repeated (threefold) loadings and, thus, is independent of the rate of deformation.

Data also have been obtained which confirm the significant role of moisture in the shaping of viscosity effects in sandy soils.

An estimate of the parameters of the distribution functions during measurements of the stresses and deformations in an equipment of the quasistatic type and an estimate of the accuracy of these measurements permit us to proceed to a quantitative estimate of the mechanical characteristics of soils, taking into account their viscoplastic properties during brief dynamic loadings.

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